PRAXES PROXIES: REVISITING EDUCATIONAL MANIPULATIVES FROM AN ECOLOGICAL DYNAMICS PERSPECTIVE

Dor Abrahamson
UC Berkeley
dor@berkeley.edu

Raúl Sánchez-García
European University, Madrid
raulsangar@gmail.com

Dragan Trninic
National Inst. of Ed., Singapore
trninic.dragan@gmail.com

The recent proliferation of technological devices with natural user interfaces (e.g., touchscreen tablets) is regenerating scholarship on the role of sensorimotor interaction in conceptual learning. Some researchers of mathematical education have adopted views from constructivism, phenomenology, enactivism, and ecological dynamics to interpret implicit sensorimotor schemes as both forming and manifesting disciplinary competence. Drawing on these views, this theoretical paper returns to the enduring question of what it means to develop a new skill by way of task-oriented interaction with objects. Beginning with sports then moving to mathematics, we focus on a subcategory of pedagogical artifacts that serve students only during training activities yet constitute proxies for developing target schemes toward normative application. We argue for the contribution of these views to conceptualizing the design of effective mathematics instruction.

Keywords: Instructional Activities and Practices, Learning Theory, Technology

Background and Motivation: Teaching for Learning Mathematics as an Activity

Hans Freudenthal (1971, p. 435) once remarked that learning mathematics is better understood as learning how to swim. This remark could be understood as metaphorical, that is, Freudenthal is indicating that learning mathematics is an *activity*, rather than something passively done to the student. And yet one way to teach mathematics as an activity is to have students *physically* engage in the mathematical process, much like swimming necessarily involves an initial period of flailing one's arms and legs in water.

This theoretical paper is based on the speculative premise that learning mathematics is literally, not only metaphorically, much like learning to swim. Accordingly, we will be devoting the first portion of this paper to introducing, from the sports science, a framework for modeling how people learn to perform physical techniques, such as martial-arts maneuvers. Only once we have established this analytical model will we then turn to discuss the case of mathematics, which we perceive as significantly analogous.

To allay the reader's concern, let us stress that this project is not construed naively: We are well aware of patent differences between sports and mathematics. To an onlooker, freestyle swimming appears quite different from solving for *x*. The overt physical actions, tasks, goals, media, and semiotic register and tenor across these disparate cultural practices are indeed incongruous. And yet we submit that these apparent differences need not undermine a project to investigate for meaningful similarities across the activities.

A comprehensive comparison of sports and mathematics is probably beyond the scope of this modest paper. And so our tack here will be to carry out the comparison by way of selecting a particular prism as a means of narrowing and focusing the comparison. We have chosen to examine what it means to learn a skill with the aid of an object, what is often referred to in the mathematics-education literature as a manipulative. We are thus proposing here an inter-disciplinary comparative analysis of the didactical function of manipulatives in sports and in mathematics, because we believe that this comparison bears potential rewards for the theory, design, and practice of mathematics education more generally.

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

Praxes Proxies: Pedagogical Artifacts for Practicing Novices

To even further hone this paper, we will look at an exclusive class of manipulatives, those that are used not in enacting an actual goal practice in its normative trappings but only in training toward enacting that goal practice. For example, a soccer player might develop prowess by way of dribbling in slalom path along a line of cones, even though there would never be cones on the field during an actual game. Here the cones are standing in for people, and they are positioned in a form that exaggerates and distils the core coordination challenge facing the novice.

We are tentatively calling this class of objects *praxes proxies*, because we believe they do not quite fall squarely under any current definition of materials or activities that are employed in the service of learning a new skill. For example, one might think of this specialized class of artifacts as scaffolds, in the sense that they constitute auxiliary elements or structure temporarily deployed into a learning environment and then later removed (faded) once the learners are prepared to engage in normative practice, as in the case of bicycle trainer wheels (Bakker, Smit, & Wegerif, 2015). And yet our artifacts of interest are not really scaffolds, because they are used not only to support the student's physical execution of otherwise overwhelming actions but also at times to simulate ecologically authentic environmental features actually encountered in normative experiences—our artifacts of interest might *pose* rather than preempt problems. One might therefore think of our artifacts of interest as enabling what Kirsh (2010) would view as a form of rehearsal, in the sense that they support practitioners in reducing, decomposing, or schematizing a complex practice by way of focusing on some circumscribed aspect. And yet these artifacts actually supplement structure onto the activity—structure that does not exist in the goal activity. Moreover, the student has not yet experienced the goal practice and thus cannot have any agency in decomposing it. One might therefore think of these artifacts as enabling a form of simulation of the goal activity that is less dangerous, costly, or otherwise resource consuming (Schwartz, 2007). And yet we are looking at cases where the learner need not or even cannot be aware of how the performance of these unique exercises is related to the goal practice, so that it does not make sense to call them simulations, at least not in view of the student's own phenomenology.

By calling our focal class of pedagogical objects *praxes proxies* we are considering them as basic-training substitutes for critical aspects of "the real thing." As we will be arguing, praxes proxies serve to entrain learners toward operating effectively within target activity contexts by way of creating for the students opportunities to develop sensorimotor coordinations relevant to, and applicable in enacting the goal practice. These new sensorimotor coordinations are elicited from students as they attempt to overcome performance challenges that emerge in the course of satisfying a task objective. From the systemic perspective that we will be evaluating, learning is developing new sensorimotor coordination oriented on newly constructed aspects of the environment: As they practice, students reconfigure the environment to afford new interactions.

Our focus herein on praxes proxies could be instrumental in considering educational phenomena. Namely, praxes proxies are quite unique in that they have been historically designed or selected explicitly for teaching and learning a skill, not for actually performing the skill in its goal contexts. The very existence of this class of objects should be interesting to educational researchers, because it helps us isolate and thus examine "purely" pedagogical affordances of cultural practice as they relate to the target skills they foster. The sections below clarify and demonstrate our thesis, arguing for its broad utility.

Theoretical Framework

Why might researchers of mathematics education care how people learn to perform sports techniques? This intellectual orientation warrants some explaining.

We grant that the learning of physical skills is often seen as substantially different from the learning of conceptual notions. Yet we ask the reader to keep an open mind, as looking to physical

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

disciplines for inspiration may not be as radical as it appears at first glance. Indeed, over thirty years ago, in his address to the International Group for the Psychology of Mathematics Education, when von Glasersfeld (1983) criticized research in mathematics education for having under-delivered, he added,

> this disappointment—I want to emphasize this—is not restricted to mathematics education but has come to involve teaching and the didactic methods in virtually all disciplines….There is only one exception that forms a remarkable contrast: the teaching of physical and, especially, athletic skills. There is no cause for disappointment in that area. (p. 42)

von Glasersfeld argued that we ought to learn from those physical domains that have been exceptional, because

> the primary goal of mathematics instruction has to be the students' conscious understanding of what he or she is doing and why it is being done…[W]hat the mathematics teacher is striving to instill into the student is ultimately the awareness of a dynamic program and its execution—and that awareness is in principle similar to what the athlete is able to glean….from his or her performance. (pp. 51-52)

We agree with von Glasersfeld that analogizing competence in sports and mathematics might go more than skin deep. Our work has been an attempt to implement this radical-constructivist epistemological position in the form of pedagogical activities designed for mathematics students to ground targeted curricular content in new forms of physical activity they learn to enact. Accordingly, in this paper we consider content learning as sprouting from 'dynamic programs' then maturing via guided reflection into 'conscious understanding.' The heart of this paper is on artifacts that both elicit and shape said dynamic programs. Yet just before we focus on these artifacts, we explain how our epistemological position on grounded mathematics is not as arcane as might at first appear.

This paper is situated within a current turn in mathematics-education research toward theorizing conceptual knowledge as grounded in sensorimotor schemes. Per this view, learning new concepts requires the development of new spatial–dynamical motor-action coordinations. This notion, which is shared by cognitive developmental psychology (Piaget, 1968), enactivism (Varela, Thompson, & Rosch, 1991), and neuro-educational research (Norton & Deater-Deckard, 2014), orients mathematics education researchers on the action patterns themselves—how a physical task is accomplished—equally or perhaps more so than on the material or semiotic products of these actions (Nemirovsky, 2003). We hope with this paper to contribute to the field's discourse specifically around designing activities that foster targeted action patterns.

Several reasons account for our somewhat unusual program of research that has led us to collaborate with scientists who specialize in modeling how people develop and control motor action. To begin with, converging reports from empirical research studies in diverse branches of the cognitive sciences have been putting forth claims to the effect that cognitive activity is grounded in the tacit enactment of perceptually guided physical motor action, even when no overt corporeal action is manifest to the on-looking observer (Barsalou, 2010). In fact, it has been claimed that sports psychology is essential for understanding cognition (Beilock, 2008). It thus stands to reason, at least per our judgment, that researchers of mathematical cognition should have a firm grasp of the relation between sensorimotor and mathematical activity (see Nemirovsky, 2003).

Granted, the thesis that concepts are grounded in sensorimotor action is foundational to constructivism (Piaget, 1968) and carries through to neo-Piagetian scholarship (Kalchman, Moss, & Case, 2000; Norton & Deater-Deckard, 2014). And yet, by-and-large the community of mathematics-education researchers is not equipped to capture, document, analyze, and model these sensory perceptions and physical motor actions in which concepts are allegedly grounded (Abrahamson & Sánchez–García, 2015, in press). For example, research articles on mathematics learning process

rarely offer micro-ethnographic analyses of the minute sensorimotor schemes students develop via solving interaction problems. When articles do offer these analyses, we witness careful transcriptions of multimodal activity that include descriptions of students' and teachers' instrumental and representational gestures (Nemirovsky & Ferrara, 2009). However, these descriptions are not framed by, and therefore do not attend to, kinesiological properties of these actions. That is, the studies might espouse an embodied-cognition framework to explain why a concept is challenging and how teachers and students interact with artifacts to construct and elucidate the inherent mathematical notions, yet the studies are not founded on the premise that *the physical actions themselves are challenging and that this challenge is explanatory of the conceptual challenge of grasping and applying the new notions*.

To be sure, lifting a fist-sized red block and placing it on a nearby blue block is certainly within the motor skills of normally developing kindergarten students. For this population, the physical motor operation of re-positioning a block is not designed in and of itself to be challenging, and so performing this rudimentary isolated physical operation per se is not conceived as fostering the development of a new sensorimotor scheme. Rather, to the extent that we view mathematics learning as contingent on overcoming sensorimotor challenges, this view is situated in a relatively new design genre for STEM education, namely in technologically enhanced embodied learning environments (TEELE, see Lindgren & Johnson-Glenberg, 2013).

Pioneered by Nemirovsky and his collaborators (e.g., Nemirovsky, Tierney, & Wright, 1998), TEELE involve a dynamical interaction task that turns out to demand a challenging sensorimotor coordination. Performing the task is enacting a new form of reasoning. Further classroom discourse on the solution, which may elicit reflection, description, representation, interpretation, and argumentation, makes to consolidate the new form of reasoning in normative semiotic registers, such as vocabulary, diagrams, graphs, and symbolic notation (Abrahamson & Lindgren, 2014). It is these environments—and in particular their central focus on dynamical solutions to sensorimotor coordination problems—that have been motivating our research program to step aside from mainstream mathematics-education scholarship and seek the wisdom and practice of disciplines dedicated to the investigation of motor-action learning and control.

In sum, we are interested in a class of cultural artifacts we call praxes proxies. Unique about these artifacts is that they are not used in performing some goal practice per se, but rather to *train* towards performing this goal practice. For example, consider the speedbag used in boxing. This artifact is viewed as invaluable for training boxers yet is clearly *unlike* what is enountered in an actual boxing match. As von Glasersfeld stipulates, the pedagogical utility of these artifacts is that through training students become aware of, or attuned to, some aspect of practice. We argue that mathematics education, particularly the design of manipulatives, stands to benefit from leveraging what other fields have discovered about the pedagogical utility of these artifacts.

In the remainder of this paper we turn directly to an emerging theoretical framework from sports sciences, ecological dynamics, so as better to understand the contribution of praxes proxies to students' development of competence in a physical activity. We will then illustrate an analogous case from mathematics education.

Ecological Dynamics & Non-Linear Pedagogies: Introducing Adequate Constraints

Relations between student, task, and available artifacts can be modeled after an ecological-dynamics perspective (Abrahamson & Sánchez–García, 2015). From this view, the student, task, and artifacts comprise a system. As the student attempts to perform some goal task, we say that the artifacts constitute productive constraints on these efforts. By "productive constraints" we mean just that—the artifacts usefully negate a vast spectrum of possible yet culturally irrelevant routes of action (degrees of freedom) leaving only a narrow range of possible ways to go about completing the task, where these ways are in line with the pedagogical intention of the activity. A child building a

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

castle from interlocking plastic blocks will learn through exploration to adjoin the blocks according to the designer's intention: For this child, the blocks will come to privilege (afford) particular modes of interacting. As researchers, we look to understand, build, and evaluate learning environments that foster new dynamical interaction patterns by productively constraining how a learner might engage an activity in seeking to satisfy some task objective.

Ecological dynamics originates in the sports sciences. The application of dynamical systems to ecological psychology enables sports scientists to explain the learning of physical activities as the complex self-organizing of subject–environment dynamical systems (Vilar, Araújo, Davids, & Travassos, 2012). In this systemic approach, learning is modeled not as generating a sequence of disembodied symbolical propositions, such as abstracted inferences and decisions, but as intrinsically emergent from and tuned to the agent's embedded action structures within a non-linear system (Araújo, Davids, Chow, Passos, & Raab, 2009). Thus, the unit of analysis is not the isolated individual but the indissoluble pair of individual–environment in interaction.

The self-organizing behavior of this agent–environment system can be affected or "channeled" by different kinds of constraints. Newell (1996) identified three sources of constraints affecting the behavior of the system: organismic (biochemical, biomechanical, neurological), environmental (gravity, temperature, light), and task (goals and rules). In this paper we are theorizing the role of supplementary artifacts as introducing task constraints appropriate to the pedagogical objectives. Notably, the introduction of praxis-proxy artifacts changes the task, and so we may consider them as task constraints.

From a didactical point of view, the introduction of adequate constraints becomes a paramount issue. Non-linear pedagogy (Chow et al., 2011) is based on supplementing and modifying constraints in the learning environment. Coaches, similar to constructivist mathematics educators, adopt a strategy of discovery-based learning, where students' learning process is constrained in deliberate ways. By assigning what should be done and constraining how it might be accomplished, instructors generate "fields of promoted action" (Reed & Bril, 1996). Therein, learners are encouraged to engage in explorative behavior by which to find personal solutions to the task at hand. Such constraints can be introduced either by changing the game's rules/conditions, changing/restricting physical space, modifying equipment, increasing gradually the complexity of the task, or simplifying it (Davids et al., 2008, pp. 161-167). The main aim of this cluster of activities is to foster self-discovery by providing enough variability for individual learners to find their own solutions to varying situations that bear for the learners emergent contingency (Bernstein, 1967).

The introduction of constraints must always follow the principle of "representative design" (Brunswik, 1956): Activities created specifically for training purposes should not distort or change some key information of the environment that learners would find in the actual conditions of the real game (Renshaw, Davids, Shuttleworth & Chow, 2009). Rather, learners are to engage in behaviors that foster attunement towards the key perceptual information sources pertaining to the ultimate physical performance in the authentic goal context. Chow et al. (2009) stress that practice activities must be representative of performance demands so as to lead to transfer of skills between practice and performance environments.

Artifacts that create representative-design task constraints might be either complex or simple and even mundane familiar objects. Consider a well-familiar object: a wall (see Figure 1).

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

Figure 1. In Systema, a Russian self-defense method, a wall serves in an exercise, per the principle of representative design. Stand opposite a wall. Lay your hands on it. Now "walk" your hands down the wall, one hand at a time. All the while, your feet, too, must walk backwards so as to maintain a more-or-less straight body. Once low, you would walk your hands into a push-up position on the floor. You would then reverse the sequence back up to the starting position.

The introduction of this simple artifact (a wall) channels your activity into dynamically maintaining a corporeal structure able of sustaining a line of force transmitted from feet to hands, thus resulting in a constant pushing-forward action. Such performance is crucial when facing a fighting opponent who pushes you backwards with her attack. Thus, the wall exercise fosters the student's attunement to key information coming from the haptic (dynamic touch) sense. The wall acts as a praxis proxy, substituting an opponent yet maintaining representative-design principles.

We thus view praxes proxies in terms of their systemic role. As students attempt to perform the given task under the constraints poised by the artifact, the students learn to move in new ways. That is, the environment comes to afford new ways of moving, namely new motor-action coordinations. (By way of cultural reference, the reader might remincise about the Karate Kid.)

Praxes proxies are a unique class of pedagogical artifacts in the sense that they serve students in the deliberate absence of actual goal contexts, creating intact worlds that idealize or essentialize specified aspects of the normative tasks. Various gym devices serve in this capacity by way of demarcating a space and dedicated equipment for engaging in an activity that, to the naked eye, bears little to no ecological authenticity (e.g., we rarely waddle down a wall). Yet this activity nevertheless focuses attention and effort on developing and exercising a targeted corpus of motor action highly relevant and environmentally attuned to ecologically authentic tasks.

As we turn from sports to math, we stress that across these domains praxis proxies

> foster not only local skill development per se, that is, becoming better at some particular movement, but are methods for gaining insight into disciplinary knowledge. In other words, the central argument is that, through their practice, these routines serve as *methods* of developing disciplinary knowledge. [They are]….not practiced for their own sake, but for what is gained through practice. (Trninic, 2015, p. 24)

Praxis Proxies in Mathematics Education: The Mathematical Imagery Trainer

From the epistemological perspective of embodiment theories, we have argued, learning mathematical and athletic skills is similar, in that both require the construction of new sensorimotor schemes as a condition for competent performance. Creating representative design for mathematics learning hinges on determining, perhaps inventing, particular sensorimotor schemes that arguably capture the dynamical cognitive substrate of reasoning about and toward a target concept. Thus, if we the designers wish to create praxes proxies for proportional reasoning, we must first account for what proportional reasoning looks like, feels like, moves like for us—we must 'phenomenalize' (Pratt & Noss, 2010) proportional reasoning in the form of a sensorimotor coordination and then build embodied-interaction tasks that foster the development of these very coordinations by way of solving some performance challenge.

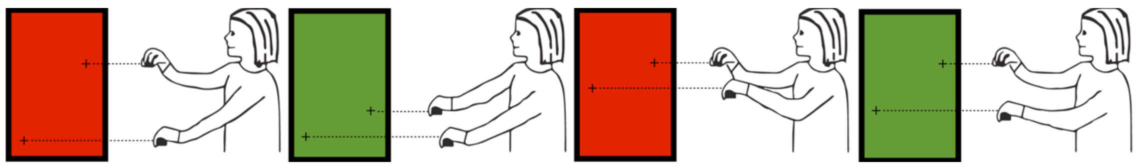

Figure 2. The Mathematical Imagery Trainer for Proportion (MIT-P) set at a 1:2 ratio, so that the favorable sensory feedback (a green background) is activated only when the right hand is twice as high from the monitor's bottom line as the left hand. This figure sketches out our Grade 4 – 6 study participants' paradigmatic interaction sequence toward discovering and then practicing an effective operatory scheme: (a) while exploring, the student first positions the hands incorrectly (red feedback); (b) stumbles upon a correct position (green); (c) raises hands maintaining a fixed interval between them (red); and (d) corrects position (green). Compare 1b and 1d to note the different vertical intervals between the hands and, correspondingly, between the virtual objects.

An example for a mathematics praxis proxy is the Mathematical Imagery Trainer for Proportion (see Figure 2; Abrahamson & Howison, 2010). We first designed a bimanual motor-action scheme that enacts proportional equivalence, and then we engineered conditions for students to move in a new way that would require developing this scheme. Our two-step activity plan was for students to: (1) develop a target motor-action scheme as a dynamical solution to a situated problem bearing no mathematical symbolism; and (2) describe these schemes mathematically, using semiotic means we then interpolate into the action problem space.

Empirical evaluation of the design, including the integrated micro-genetic analysis of clinical, electronic, and eye-tracking data, suggests that indeed students are devising target sensorimotor schemes in solving the interaction problems (Abrahamson et al., 2016).

Conclusion

We are only just beginning to understand the relation between bimanual coordination and mathematical cognition. Researching the emergence of this digital-cum-digital prowess, from manual action to symbolic notation, is contingent upon thinking out of the curricular box to design opportunities for witnessing and investigating this emergence. And yet for all these efforts ultimately to contribute to education in its authentic ecology, we must strive to remove borders between research and practice. Per the PME-NA 38 call, "We see borders as potentially productive as well as potentially problematic." We agree: We seek partners across the border to engage in productive, synergistic discourse so that mathematics learning can move in new ways.

References

Abrahamson, D., & Howison, M. (2010, May). *Embodied artifacts: Coordinated action as an object-to-think-with*. In D. L. Holton (Organizer & Chair) & J. P. Gee (Discussant), Embodied and enactive approaches to instruction: implications and innovations. Paper presented at the Amer. Ed. Res. Assoc., April 30 - May 4.

Abrahamson, D., & Sánchez-García, R. (2015). A call to action: Towards an ecological-dynamics theory of mathematics learning, teaching, and design. In T. G. Bartell, K. N. Bieda, R. T. Putnam, K. Bradfield, & H. Dominguez (Eds.), *Proceedings of the 37th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 12, pp. 1261-1268). East Lansing: MSU.

Abrahamson, D., & Sánchez-García, R. (in press). Learning is moving in new ways: The ecological dynamics of mathematics education. *Journal of the Learning Sciences*. doi:10.1080/10508406.2016.1143370.

Abrahamson, D., Shayan, S., Bakker, A., & Van der Schaaf, M. F. (2016). Eye-tracking Piaget: Capturing the emergence of attentional anchors in the coordination of proportional motor action. *Human Development, 58*(4-5), 218-244.

Abrahamson, D., & Lindgren, R. (2014). Embodiment and embodied design. In R. K. Sawyer (Ed.), The Cambridge handbook of the learning sciences (2nd ed.) (pp. 358-376). Cambridge, UK: Cambridge University Press.

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

Araújo, D., Davids, K. W., Chow, J. Y., Passos, P., & Raab, M. (2009). The development of decision making skill in sport: An ecological dynamics perspective. In D. Araújo & H. Ripoll (Eds.), *Perspectives on cognition and action in sport* (pp. 157-169). Hauppauge, NY: Nova Science.
Bakker, A., Smit, J., & Wegerif, R. (Eds.) (2015). Scaffolding and dialogic teaching in mathematics education. *ZDM Mathematics Education, 47*(7).
Barsalou, L. W. (2010). Grounded cognition: Past, present, and future. *Topics in Cognitive Science, 2*(4), 716-724.
Beilock, S. L. (2008). Beyond the playing field: Sport psychology meets embodied cognition. *International Review of Sport and Exercise Psychology, 1*(1), 19-30.
Bernstein, N. A. (1967). *The control and regulation of movements*. London: Pergamon Press.
Brunswik, E. (1956). *Perception and the representative design of psychological experiments*. Berkeley: UCP.
Chow, J. Y., Davids, K., Button, C., Shuttleworth, R., Renshaw, I., & Araújo, D. (2007). The role of nonlinear pedagogy in physical education. *Review of Educational Research, 77*(3), 251-278.
Chow, J. Y., et al. (2011). Nonlinear pedagogy. *New Ideas in Psychology, 29*(2), 189–200.
Davids, K., Button, C., & Bennet, S. (2008). *Dynamics of skill acquisition*. Champaign, IL: Kinetics.
Freudenthal, H. (1971). Geometry between the devil and the deep sea. *ESM, 3*(3/4), 413-435.
Kalchman, M., Moss, J., & Case, R. (2000). Psychological models for the development of mathematical understanding. In S. Carver & D. Klahr (Eds.), *Cognition and Instruction* (pp. 1-38). Mahwah: LEA.
Kirsh, D. (2010). Thinking with the body. In S. Ohlsson & R. Catrambone (Eds.), *Proc. of the 32nd Annual Meeting of the Cognitive Science Society* (pp. 2864-2869). Austin, TX: Cognitive Science Society.
Lindgren, R., & Johnson-Glenberg, M. (2013). Emboldened by embodiment: Six precepts for research on embodied learning and mixed reality. *Educational Researcher, 42*(8), 445-452.
Nemirovsky, R. (2003). Three conjectures concerning the relationship between body activity and understanding mathematics. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 27th Annual Meeting of PME* (Vol. 1, pp. 105-109). Columbus, OH: Eric.
Nemirovsky, R., & Ferrara, F. (2009). Mathematical imagination and embodied cognition. *Educational Studies in Mathematics, 70*(2), 159-174.
Nemirovsky, R., Tierney, C., & Wright, T. (1998). Body motion and graphing. *Cog. & Ins., 16*(2), 119-172.
Newell, K. M. (1996). Change in movement and skill: Learning, retention, and transfer. In M. L. Latash & M. T. Turvey (Eds.), *Dexterity and its development* (pp. 393-429). Mahwah: LEA.
Norton, A., & Deater-Deckard, K. (2014). Mathematics in mind, brain, and education: A neo-Piagetian approach. *International Journal of Science and Mathematics Education, 12*(3), 647-667.
Piaget, J. (1968). *Genetic epistemology* (E. Duckworth, Trans.). New York: Columbia University Press.
Pratt, D., & Noss, R. (2010). Designing for mathematical abstraction. *Int. J. Comp. for Math. Learn, 15*(2), 81-97.
Reed, E. S., & Bril, B. (1996). The primacy of action in development. In M. L. Latash &M. T. Turvey (Eds.), *Dexterity and its development* (pp. 431-451). Mahwah, NJ: Lawrence Erlbaum Associates.
Renshaw, I., Davids, K., Shuttleworth, R., & Chow, J. Y. (2009). Insights from ecological psychology and dynamical systems theory can underpin a philosophy of coaching. *Int. J. of Sports Ps., 40*(4), 540-602.
Schwartz, J. L. (2007). Models, simulations, and exploratory environments. In R. A. Lesh, E. Hamilton, & J. J. Kaput (Eds.), *Foundations for the future in mathematics education* (pp. 161-172). Mahwah: LEA.
Trninic, D. (2015). *Body of knowledge: Practicing mathematics in instrumented fields of promoted action*. Unpublished doctoral dissertation. University of California, Berkeley.
Varela, F. J., Thompson, E., & Rosch, E. (1991). *The embodied mind*. Cambridge, MA: M.I.T. Press.
Vilar, L., Araújo, D., Davids, K., & Travassos, B. (2012). Constraints on competitive performance of attacker-defender dyads in team sports. *Journal of Sports Sciences, 30*(5), 459-469.
von Glasersfeld, E. (1983). Learning as constructive activity. In J. C. Bergeron & N. Herscovics (Eds.), *Proceedings of the 5th Annual Meeting of PME* (Vol. 1, pp. 41-69). Montreal: PME-NA.